1. A box spread is a combination of a bull spread composed of two call options with strike prices and and a bear spread composed of two put options with the same two strike prices.
2. Describe the payoff from a box spread on the expiration date of the options.
3. What would be a fair price for the box spread today? Define variables as necessary.
4. Under what circumstances might an investor choose to construct a box spread?
5. What sort of investor do you think is most likely to invest in such an option combination, i.e. a hedger, speculator or arbitrageur? Explain your answer.

2. Form a long butterfly spread using the three call options in the table below.

|  |  |  |  |
| --- | --- | --- | --- |
|  | **C1**  **X = $90**  **T = 180 days** | **C2**  **X = $100**  **T = 180 days** | **C3**  **X = $110**  **T = 180 days** |
| Price | 16.3300 | 10.3000 | 6.0600 |
| DELTA | 0.7860 | 0.6151 | 0.4365 |
| GAMMA | 0.0138 | 0.0181 | 0.0187 |
| THETA | -11.2054 | -12.2607 | -11.4208 |
| VEGA | 20.4619 | 26.8416 | 27.6602 |
| RHO | 30.7085 | 25.2515 | 18.5394 |

1. What does it cost to establish the butterfly spread?
2. Calculate each of the Greek measures for this butterfly spread position and explain how each can be interpreted.
3. How would you make this option portfolio delta neutral? What would be achieved by doing so?
4. Suppose that tomorrow the price of C1 falls to $12.18 while the prices of C2 and C3 remain the same. Does this create an arbitrage opportunity? Explain.
5. Consider a six month American put option on index futures where the current futures price is 450, the exercise price is 450, the risk-free rate of interest is 7 percent per annum, the continuous dividend yield of the index is 3 percent, and the volatility of the index is 30 percent per annum. The futures contract underlying the option matures in seven months. Using a three-step binomial tree, calculate
6. the price of the American put option now,
7. the delta of the option with respect to the futures price,
8. the delta of the option with respect to the index level, and
9. the price of the corresponding European put option on index futures.
10. Apply the control variate technique to improve your estimate of the American option price **and** of the delta of the option with respect to the futures price.

Note that the Black-Scholes price of the European put option is $36.704 and the delta with respect to the futures price given by Black-Scholes is –0.442.

4. A financial institution trades swaps where 12 month LIBOR is exchanged for a fixed rate of interest. Payments are made once a year. The one-year swap rate (i.e., the rate that would be exchanged for 12 month LIBOR in a new one-year swap) is 6 percent. Similarly the two-year swap rate is 6.5 percent.

1. Use this swap data to calculate the one and two year LIBOR zero rates, expressing the rates with continuous compounding.
2. What is the value of an existing swap with a notional principal of $10 million that has two years to go and is such that financial institution pays 7 percent and receives 12 month LIBOR? Payments are made once a year.
3. What is the value of a forward rate agreement where a rate of 8 percent will be received on a principal of $1 million for the period between one year and two years?

Note: All rates given in this question are expressed with annual compounding.

1. The term structure is flat at 5% per annum with continuous compounding. Some time ago a financial institution entered into a 5-year swap with a principal of $100 million in which every year it pays 12-month LIBOR and receives 6%. The swap now has two years eight months to run. Four months ago 12-month LIBOR was 4% (with annual compounding). What is the value of the swap today? What is the financial institution’s credit exposure on the swap?
2. An American put option to sell a Swiss franc for USD has a strike price of 0.80 and a time to maturity of 1 year. The volatility of the Swiss franc is 10%, the USD interest rate is 6%, and the Swiss franc interest rate is 3% (both interest rates continuously compounded). The current exchange rate is 0.81. Use a three time step tree to value the option.
3. A European call option on a certain stock has a strike price of $30, a time to maturity of one year and an implied volatility of 30%. A put option on the same stock has a strike price of $30, a time to maturity of one year and an implied volatility of 33%. What is the arbitrage opportunity open to a trader. Does the opportunity work only when the lognormal assumption underlying Black-Scholes holds. Explain the reasons for your answer carefully.
4. A put option on the S&P 500 has an exercise price of 500 and a time to maturity of one year. The risk free rate is 7% and the dividend yield on the index is 3%. The volatility of the index is 20% per annum and the current level of the index is 500. A financial institution has a short position in the option.

a) Calculate the delta, gamma, and vega of the position. Explain how they can be interpreted.

b) How can the position be made delta neutral?

c) Suppose that one week later the index has increased to 515. How can delta neutrality be preserved?

1. An interest rate swap with a principal of $100 million involves the exchange of 5% per annum (semiannually compounded) for 6-month LIBOR. The remaining life is 14 months. Interest is exchanged every six months. The 2 month, 8 month and 14 month rates are 4.5%, 5%, and 5.4% with continuous compounding. Six-month LIBOR was 5.5% four months ago. What is the value of the swap?

The Deutschemark-Canadian dollar exchange rate is currently 1.0000. At the end of 6 months it will be either 1.1000 or 0.9000. What is the value of a 6 month option to sell one million Canadian dollars for 1.05 million deutschemarks. Verify that the answer given by risk neutral valuation is the same as that given by no-arbitrage arguments. Is the option the same as one to buy 1.05 million deutschemarks for 1 million Canadian dollars? Assume that risk-free interest rates in Canada and Germany are 8% and 6% per annum respectively.